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Social Network Analysis: A brief theoretical review and further perspectives in the study of Information Technology

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ABSTRACT

Social Network Analysis is a widely used approach in psychology, as in social science, economics and other fields. The peculiarity of this perspective is that it focuses not on individuals or other social units, but on the relationship between them. In this paper, our aim is to give a general review of this perspective, giving a description of resources and principal topics covered by Social Network Analysis. In the first section, we will concentrate on methodological and formal perspectives of analysis. In the last section, we will introduce some recent studies about Social Network and its relationship with Information Technologies, in particular on the Internet. Lastly, we will show how this approach can be useful to study some aspects of the web.

Keywords: *Social Network Analysis, Social Web, blogosphere, social network indexes, social quantitative indexes, hyperlinking networks, roles algebra, Computer Mediated Communication.*

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1. Introduction

Every kind of social aggregation can be represented in terms of units composing this aggregation and relations between these units. This kind of representation of a social structure is called “Social Network”. In a social network, every unit, usually called “social actor” (a person, a group, an organization, a nation, a blog and so on), is represented as a node. A relation is represented as a linkage or a flow between these units. The set of possible relations is potentially infinite; the term relation can have many different meaning: acquaintance, kinship, evaluation of another person, the need

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of a commercial exchange, physical connections, the presence in a web-page of a link to another page and so on.

Therefore, the objects under observation are not individuals and their attributes, but the relationships between individuals and their structure. The advantage of such a representation is that it permits the analysis of social processes as a product of the relationships among social entities.

In Social Network Analysis we can study two different kinds of variables: *structural* and *composition.* Variables of the first type are the most important in this field because they represent the different kinds of ties between social actors (friendship, trust and so on). The composition variables can be seen as actors' attributes. To make a deeper attribute analysis we suggest the use of more sophisticated formal theories, like Formal Concept Analysis (Wille, 1982).

2. Main characteristics

2.1. Social Network Analysis history

According to Scott (1991), there are three main research lines about Social Network Analysis, lead respectively by:

- Sociometric Analysts, who mainly used and developed graph theory;
- Harvard researchers, who in a first moment studied models of interpersonal relationship and cliques formation, and after 1970 developed algebraic models of SNA;
- The anthropologist part of the school of Manchester, who studied relational structures characterizing tribal society community.

There is a general agreement about the origins of the Social Network Analysis: the first studies were realized by Jacob Moreno (1889-1974) on "sociometric" (1934) and by Fritz Heider (1896-1988) on the triad equilibrium analysis (1946). In recent times, some authors have criticized this idea, marking how the concept of Social Network was developed before the famous Moreno study (Freeman, 1992).

These ideas were first developed by Frank Harary (1921-2005) and Dorwin Cartwright (-1988) which, using methods from the newborn graph theory (König, 1936), found a powerful formal tool for social structures analysis (Cartwright & Harary, 1956; Harary, Norman & Cartwright, 1965). According mainly to Heider theories, Harary and coll. studied an extension to a more complex case of structural balancing (i.e. an application of the notion of equilibrium to groups with more than three elements). The base theory was to decompose a graph (a group) in different subgraphs (subgroups), in order to analyze relationships and balancing between and within groups: every balanced graph can be divided in subgroups with positive internal relations and negative external relations.

Around the Thirties and Forties, some Harvard researchers dedicated their attention mainly to the search for decomposition and exploration methods of blocks or structures composing a graph, and their informal relations. In particular, the two leaders of the movement were W. Lloyd Warner (1898-1970) and Eltan Mayo (1880-1949), who started several researches about Chicago's central plant "Hawthorne", employed working conditions and about New England's communities. The originality of their studies consisted of the large usage of sociograms (Mayo, 1945), and the introduction of the concept of "clique" as "intimal not-parental group" (Warner, Lunt, 1941). We must say that this notion of clique is different from the sociometric meaning typical of graph theory, which we will discuss in section 3.2: Warner described a clique as some kind of "social set" (Scott, 1991).

Moreno's sociometry and Warner and Mayo's theories were unified by a Harvard professor, George Homans (1910-1989), who thought sociometry to be a good and valid foundation for social analysis (Homans, 1951).

A new and fundamental development in SNA was due to researchers from Manchester University Department of Social Anthropology. Manchester researchers pointed their attention at the effective configuration of relationships deriving from power and conflict between individuals, instead of set up norms and institutions of a society (Scott, 1993). John Barnes (1918-), who first introduced the term "Social network" (Barnes, 1954), gave life to a remarkable formal development of the analysis of social structures. Developing Barnes and colleagues' ideas, Siegfried Nadel (1903-1956) began his fundamental works, underlying the importance of a structural analysis rather than a contents analysis (Nadel, 1957). The main topic of his works, unfortunately interrupted by his premature death, was the notion of "role", considered as the foundation of social life, and defined by the interrelation of independent networks. Nadel also suggested the utility of algebraic tools for role analysis; although he had no opportunities to develop such a proposal, we must say that this kind of approach is central and of great influence in current studies (Pattison, 1993).

Nadel's ideas were pursued by Clyde Mitchell (1918-1995), which introduced the difference between total network and ego-centred or local network (see section 2).

Mitchell's analysis is important because of the reflection about some social indexes (as density, see section 3.2) and their meaning in the description of a network too.

In the following years, new important theoretical and formal acquisitions were developed in Harvard: in the setting of a continual development of algebraic models, the idea of "block model" was introduced (see section 4).

As suggested by Nadel (1957), the concept of role began to take a central part in Social Network Analysis: many ideas were developed by Harrison C. White and his colleagues.

White's group focused on the analysis of mathematical-formal aspects of Social Network, giving new directions to graph theory and introducing some new important characteristics coming from semigroup algebra (see section 4), with the aim to formalize different structural relations inside a group (White, 1963).

As we said, they introduced the concept of block model, which was intended as corresponding to the role of a group of components of a social network (Lorrain & White, 1971; White, Boorman & Breiger, 1976). The central idea of White is that the search of structures in a network should not be based on a-priori defined and well-known categories, but on the real relations among the network nodes and on how these relations structures it, with the aim to describe the concrete and emergent role structures.

In this context, a study by Mark Granovetter on the importance of weak ties is really interesting (Granovetter, 1973). His idea was shown to be useful in many researches, as in his analysis of looking for a job mechanisms (Granovetter, 1974).

From the Harvard group, a former student of White's, Barry Wellman (1978), created an international group aimed to get SNA researchers together. This group was called INSNA (an acronym for International Network Society of Social Network Analysts); its web page is available at the following URL: http://www.insna.org/INSNA/insna_inf.html.

Among the different kinds of analysis, an original theory, known as "small world phenomenon" and proposed by Stanley Milgram (1933-1984), revealed to be rich of cues. His approach concerns the empirical effort to determine how many steps (or ties) are necessary, in a well-defined population, so that two different individuals can meet each other. For instance, given a source individual that we call *a*, the number of individuals tied by a relation of acquaintance so that a chain that links *a* to a different target individual *b* is calculated. Notwithstanding the large number of methodological criticisms posed to the original experimental paradigm, the problem turned out to be of remarkable general interest, and the "Small world phenomenon" has became a large studied issue; in particular, many studies have been carried out in order to explore which factors determine the shortest path search in a network between two individuals, and how a defined system could be defined a "small world" or not. An empirical approach of this phenomenon has been studied in relation to the construction of virtual network on the web, giving birth to system like Friendster, or to explore the functioning of Instant Messaging software (Adamic & Eydar, 2005).

We want to end this section pointing out Social Network Analysis interdisciplinary endeavour: it was born from the joint activity of social psychologists, anthropologists, sociologists, mathematicians, physicians and economists, and can be used with profit for research in behavioral, social, economical or political discipline. In the following sections we will show how mathematics (in particular, algebraic methods and graph theory) has also a strong tie with SNA. We want to point out that this tie is not monodirectional: empirical applications of graph theory had been proven to be a benefit for the graph theory itself, because this application gave the possibility to give life to some new ideas and to develop some aspects of this branch of the mathematic (Harary was a mathematician, and his innovative ideas were often used to get the graph theory richer).

A critique seldom posed to Social Network Analysis concerns the possible triviality of the results: Boissevain (1979) pointed out how SNA is mainly focused on technical issues, rather than empirical research about social topics. On the contrary, Barnes and Harary (1983) answer to this critique arguing how graph theory is a powerful and not still used at its full capability perspective. Researchers have made too little a use of this theory and its full potentiality:

> *Graph theory uses two primitive, undefined terms, point and line; these two terms are mentioned in a small number of axioms, unproved statements assumed to be true. [...] Its theorem consists of statements each of which can be derived logically either directly from the axiom system or indirectly by making use of theorems already proved. [...] (A theorem) can be used with reference to any appropriate mathematical model of the real world that has been constructed with material from its axiom system. It than reveals real world implications of the model that might otherwise have not been noticed or utilized by the designer of the model. (p.239)*

With this statement, the two authors try to underline how this potentiality is not limited to the use of a precise terminology, but the use of mathematical theories in this field can give new ideas, can *"enable to move ahead faster"*, while these methods are generally neglected. In Barnes and Harary's article, some examples are shown, and some ideas from near topics are described.

2.2. Types of studies

According to Pattison (1993), the range of use of the idea of social network is very comprehensive, but there is a general agreement about the presence of two main classes of conceptions about the role of social networks. The main idea in the first class of studies is based on the link between properties and behaviors of an individual and the environment in which he is located. In a first kind of research, SNA has been used as a tool to explain individual behavior (Wellman, 1988; Granovetter, 1985; Anderson & Jay, 1985). In a second kind of studies of this kind, it has been proposed to use the structure of the relationship of the member of a group in order to understand the social (collective) behavior of a group (Laumann & Pappi, 1976).

The second class of conceptions is based on the idea that an understanding of network structure allows the understanding of social processes occurring on that network: "social networks define paths for the flow of social traffic" (Pattison, 1993, p.4). In a first kind of studies, the nature of social networks has been studied as a function of social variables (occupation, gender, urbanizations and so on) (Blau, 1977; Coates, 1987). A similar idea is the study of the relationship between large populations parameters (for example, those connected to job finding), and some network characteristics (Granovetter, 1974), in order to explain macro-behaviors as the result of aggregations of micro-interactions among individuals (Skog, 1986).

2.3. Resources

The growing significance of Social Network Analysis research, since 1970, is well attested by the great number of papers and books connected to it. One of the main resources is the "Social Networks" journal, published by Elsevier Science, and edited by Linton Freeman and Ron Breiger, two of the most important researchers on this topic. The journal deals with theoretical, methodological and substantive themes, and is characterized by a multidisciplinary approach. Another journal dedicated to SNA is "REDES". It can be consulted online at URL: http://revista-redes.rediris.es, as "The Journal of Social Structure", available online at http://www.cmu.edu/joss. Also available online is "Connections", the official journal of INSNA.

INSNA also provided to create a mailing list for discussion on issues related to SNA; free subscription and information about it are available at the following url: http://www.insna.org/INSNA/socnet.html.

Many other journals often deal with Social Network Analysis: methodological problems are often approached in “Journal of Mathematical Psychology” (Pattison & Bartlett, 1982; Pattison & Wasserman, 1995; Pattison, et al., 2000, etc.), “Journal of Mathematical Sociology” (Cartwright & Harary, 1977; Doreian, 1987, etc.) and many others. A great interest is generated by the “small world” problem, as one can see by the great number of papers approaching broad themes as those published in “Nature” (e.g., Watts & Strogatz, 1998) and “Scientific American” (e.g., Watts et al., 2002).

Some manuals have been made to introduce SNA: “Social Network Analysis. Handbook”, written by John Scott (1991), offers a simple and empirical introduction to this topic. Methodological and analytical issues, with a particular attention to graph theory, role algebra and statistical tools are described in Wasserman and Faust’s manual, “Social Network Analysis: Methods and application”, written in 1994, and his complement “Models and Methods in Social Network Analysis”, written in 2005 by Carrington, Scott and Wasserman. An introduction to algebraic methods of analysis is provided by “Algebraic Models for Social Networks”, by P. Pattison (1993). Another interesting book, by S. Bornholdt and H. G. Schuster, is “Handbook of Graphs and Networks: From the Genome to the Internet” (2003); it shows the large potentialities of the approach we are talking about. A large number of brief introductions to SNA books are available online at http://www.insna.org/INSNA/books_inf.html.

2.4. Sampling and boundary specification: types of network

A network necessarily consists of a finite set of actors; this is due to two reasons: on one hand, this could be due to some restriction on the object under observation (for example, the interest of a researcher is the survey of the social structure of a small village or a classroom). On the other hand, this is mainly due to analytical requirements: so the problem is how to sample a group representative of the population, and how to set the constraints of this sample.

One proposal is to define unit set boundaries using the frequency of interaction, or intensity of ties among actors (Wasserman & Faust, 1994). According to Laumann et al. (1983; 1989), two different approaches can be used to specify boundaries in social network studies: the first one is called *realist approach*, the second one *nominalist* *approach*. In the realist approach, subjects have some kind of shared knowledge of the network as a social entity. In the nominalist approach, boundaries are defined by research aim, so actors can be separated by attributes of potential members of the network, relational properties between potential members, and activities in which actors are involved into.

Sometimes, when boundaries can not be defined at the very beginning, researchers use sampling techniques like *snowball sampling* (Goodman, 1961; Erickson, 1978) or *random nets* (Fararo & Skorovetz, 1984). A general review of Social Network sampling can be found in Frank (1988).

Frank (1979) categorizes snowball sampling like a “chain method”, designed to trace ties from a source to an end: an actor, or a little number of actors, is chosen among the set of potential actors. After this first step, every chosen actor reports on the actors to whom he has a link according to the relation under examination. The first actor is called *ego*, while actors chosen from him constitute the “first-order zone” of the network. Actors of the first order zone are now taken for reporting on the actor to whom every one of them as a tie, and so on. For every k step, k = 1, 2, 3..., we have a new order zone, called “*k-th order zone*”.

The main problem with sampling is that any procedures can alter or loose relations inside the considered group.

In literature, a distinction is often made between “Complete networks” and “ego-centered networks”. An *ego-centered network* (or *local network*) consists of a network built on the basis of a focal person, *ego*, like in snowball sampling. So, the entire network is referred to this focal person, and every relation is reported by the *ego*. The situation can be more complicated, if we consider every new link as an *ego* (this is the case of snowball sampling), or if we start from more than one *ego* to build a sample of a population. A *complete network* is built upon every node (or a sample of nodes) from a population, but any relation is considered for every node composing the network. The last distinction is fundamental in algebraic modelling, because it leads to different algebraic models (see section 4).

3. Methods of Social Network Analysis

Sections 3, 4 and 5 will provide a brief description of mathematical perspective from which Social Networks has been studied. We will first introduce the concept of set-theoretic relation, which is the basis of Social Network Relations. Later, we will describe the main ideas coming from graph theory and algebraic semigroup theory, to give a short description of methodological issues about Social Network Analysis. In section 5 we will describe some of the most recent developments in SNA: statistical models issues and Dynamic Network Analysis.

3.1.0. The concept of relation

The formal analysis of a social network can be dealt with different tools, every one of them with its own peculiarity, with its different operation. Also, every one of them can give different outcomes, and have different purposes.

There are two different, but not independent, perspectives from which Social Network Analysis has been observed in its short but rich history: on one hand there is the graph theory, on the other hand there is Algebraic Theory of Semi-Groups. Before starting to give a short description of these two kinds of approach, we will pinpoint their common base: the fundamental concept they share is the notion of relation, as it is considered from a set-theoretical point of view.

Definition 1: let's consider two sets, labeled X and Y; a *relation* between X and Y is every set of ordered couples where the first element belongs to X, and the second element belongs to Y.

3.1.1. Network description

For the sake of simplicity, we will now consider only the simplest form of network, a binary network (so, every relation involves only couples of units), on the set of units X.

There are three kinds of representation of a Social Network: the first description is the simple list of all the elements taken from the set of actors, and the list of the pairs of elements which are linked by a social relationship of some kind (see table 1).

$$A = \{(x_1, x_2), (x_2, x_1), (x_4, x_2), (x_3, x_1), (x_3, x_4), (x_4, x_3)\}$$
$$B = \{(x_1, x_2), (x_2, x_1), (x_3, x_4), (x_4, x_3)\}$$

Table 1: Two relations A and B on the same set $X = \{x_1, x_2, x_3, x_4\}$.

The second description comes from the Graph Theory: every actor is represented by a point, called *vertex* or *node* of the graph; the links defined by pairs of individuals, represented by lines between two linked points, are called *edges* of the graph (see fig. 1).

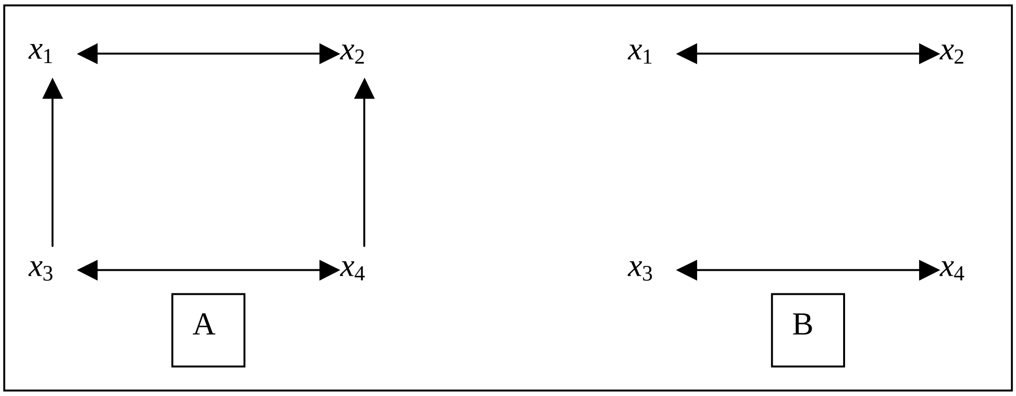

Figure 1: The corresponding graph for the relations *A* and *B* (see table 1) on the set $X = \{x_1, x_2, x_3, x_4\}$.

The third description has the form a matrix (see table 2).

	x_1	x_2	x_3	x_4
x_1	0	1	0	0
x_2	1	0	0	0
x_3	1	0	0	1
x_4	0	1	1	0

	x_1	x_2	x_3	x_4
x_1	0	1	0	0
x_2	1	0	0	0
x_3	0	0	0	1
x_4	0	0	1	0

Table 2: Corresponding Boolean matrix for *A* and *B* (see table 1) relations on the set $X = \{x_1, x_2, x_3, x_4\}$.

There are other kinds of networks: for example, there can be more than one relation, so we consider a set of different relations **R** = $\{R_1, ..., R_t\}$. Another kind of network is called “valued networks”, when a relation R_v can assume possible values different from 0 and 1). The most general binary case is that of a multiple valued network.

Definition 2: Let X represent a set of n social units, and let $v_k(i, j)$ the value of the R_k relation from unit i to unit j in X. R_k can be considered as:

23 a valued, directed graph where nodes are the elements of X and edges are the edges of type k, with value $v_k(i, j)$, directed from node i to node j;

24 a valued relation, assigning the value $v_k(i, j)$ to the ordered pair (i, j);

25 an $n \times n$ matrix, where entry (i, j) assumes the value $v_k(i, j)$.

The collection $\mathbf{R} = \{R_1,..., R_p\}$ is termed a (*multiple*) *valued network*.

Is it also possible to use relation with arity more than 2 (for example, ternary relations); this approach is not very common in Social Network Analysis: “[binary relational representations] include the vast majority of the models that have been proposed and applied in the social network literature to date” (Pattison, 1993, p. 12).

3.2. Graph Theory

Graph Theory has been used in several different scientific fields like anthropology (Hage, 1973; Zachary, 1977; Mitchell, 1980), social psychology (Heider, 1946; 1958; Leavitt, 1951; Freeman, 1977; 1979; Freeman, Reoder & Mulholland, 1980), communication (Herring et al., 2005), on-line business (Dellarocas, 2003), geography (Pitts 1965; 1979), and many others. In Social Network Analysis, Graph Theory assumes a crucial role because it has been used first of all to denote the structural properties of a network. Graph Theory works as a theoretical frame in which we can formally analyze different characteristics and properties of a network. Graph Theory is essentially a mathematical theory. For this reason it provides to Social Network Analysis a tool to quantify and measure some properties of the network. The ability to prove theorems about graphs and network structure is just a consequence of working in such a formal context. There are a lot of advantages in the use of Graph Theory in Social Network Analysis but, as it happens for many formal theories introduced in a psychological frame, often researchers prefer not to refer their studies to this formal background, so they “make too little use of the theory of graphs” (Barnes & Harary, 1983, p.235). Graph Theory, and more generally a graph, is one of the best and clearest way to represent and detect the structure defined by the way a relation (or some relations) connects different individuals of a social group. The evident advantage of the graph representation is that it provides an immediate interface through which it's possible to find, and also easily to understand, the way individuals in the network are connected.

In the following pages we'll introduce first some basic concepts of Graph Theory trying to evidence how these concepts are easily exportable to a social context. In the second part of this paragraph we'll briefly speak about some indexes we can calculate on a network using Graph Theory concepts.

3.2.1. Graphs and Di-graphs

As we have seen, the aim of Social Network Analysis is to detect and to understand the structure identified by a set of relations defined on a set of individuals. Graph Theory is an important tool for this aim. First of all we have to consider an important question: what kind of relation are we interested in to study?

We have said above that we'll investigate binary relations, so we look at only a couple of individuals a time. We have also specified that the relation is not reflexive (we don't have the relation between *x* and itself: $x \neg Rx$). But it doesn't suffice: we need to know if we are analysing a *directed* or an *undirected* relation. If the relation we analyse is undirected (relations like "to be married to…" or "to be neighbour of…" are undirected relations) we'll use a graph to represent it; otherwise (relations like "to be parent of…" or "to be subordinate to" are directed relations) we'll represent the relation using a di-graph (directed graph).

In any graph we have *g* nodes and *L* lines. We can easily find the parallelism between this formal notation and a social network: the "actors" of the network become the "nodes" of the graph, and the "ties" between actors in the network become "lines" between nodes in the graph. As we just mentioned, a graph can be used to represent an undirected relation: we can express this assumption by saying that there mustn't be any difference between a line $l_k = (n_i, n_j)$ and a line $l_h = (n_j, n_i)$ between two nodes n_i and n_j. From this sentence it derives that any undirected relation is also a symmetric relation: if n_i is married to n_j, n_j is married to n_i too.

Let us now consider the case in which we have to study a directed relation, i.e. the case in which the tie between two actors is oriented from one to the other. This condition implies that the two ties $l_k = (n_i, n_j)$ and $l_h = (n_j, n_i)$ are different. In order to represent such a relation we need to use a directed graph (*di-graph*). We define a di-graph as a set of *nodes* (the actors of the network) and a set of *direct lines* (the ties of the social network) between nodes. Every couple of nodes connected by a direct line is an ordered couple: formally $l_k= <n_i, n_j>$ means that the arc l_k is directed from the node n_i to the node n_j, and the order of this relation is not invertible (so the formula $l_k= <n_i, n_j>$ differs from the formula $l_h= <n_j, n_i>$) . The first node is called the *"sender"* while the second node is called the *"receiver".*

Now we introduce the important notion of *subgraph,* which is fundamental for several operations and indexes which we can obtain in a social network. A graph G_s is a subgraph of a graph *G* if the nodes of G_s are a subset of the nodes of *G* and the lines of G_s represent a subset of the lines of *G*.

There are some basic units of any graph that are also subgraphs: the dyad and the triad. A dyad consists of a pair of nodes and the lines (arcs) between them: in a graph we can have adjacent or not- adjacent pairs of nodes, that is to say 0 lines or 1 line; in a di-graph the situation is more composite because between any two nodes we can have 0 (*null*), 1 (*asymmetric*) or 2 (*mutual*) direct lines. Similarly a triad is composed by three nodes and the lines between them: in a graph we can have from 0 to 3 lines; in a di-graph we can have from 0 to 6 lines. Triads have been largely studied in a theoretical and practical way (e.g. Granovetter, 1973). Dyads and triads are a part of a special kind of subgraphs called *node-generated* that we are going to analyse in the next pages, together with the *line-generated* subgraph and the *cliques.*

Another useful concept is *clique*, the maximal complete subgraph composed by at least three nodes. What does maximal and complete subgraph mean? A clique is complete because each node in it is adjacent to any other node in the clique. A clique is maximal because there are no nodes not included in the clique which are also adjacent to all the nodes in the clique (Luce & Perry, 1949; Harary, Norman & Cartwright, 1965). The concept of clique introduces the larger concept of *cohesive subgroup.* A clique is the example of the most restrictive cohesive subgroup (Festinger, 1949) using graph theory. In the subsequent years several different kinds of cohesive subgroups for graphs but also for di-graphs have been investigated. Considering graphs, some of them are: *complete mutuality based subgraphs* (like cliques), *reachability and diameter based subgroups* (like *n-cliques,* see Luce, 1950; Alba, 1973; or *n-clans* and *n-clubs,* see Mokken, 1979), *nodal degree based subgroups* (like *k-plexes*, see Seidman & Foster, 1978a; or like *k-cores*, see Seidman, 1983b). Considering di-graphs some of them are *reciprocated ties based cliques* (see again Festinger, 1949; Luce & Perry, 1949) and *n-cliques for directional relations* (Peay, 1975a; 1975b; 1980).

We have presented some of the main properties of graphs relating them to Social Networks.

3.2.2. Main quantitative indexes

We will now introduce the main quantitative indexes which we can find in a network using Graph Theory.

Since the beginning of Social Network Analysis, Graph Theory has been a very important tool both to represent social structure and to calculate some indexes, which are useful to understand several aspects of the social context under analysis. In this section we present the main indexes usually associated with graphs and di-graphs.

The first and simplest index is *nodal-degree*. We define the degree of a node in a graph as the number of lines incident with that node. We can calculate the mean nodal degree of a graph with the following formula $\bar{d} = 2L/g$: we have to consider twice the total number of lines in the graph (L) because each line has to be considered both for the first and the second node. Dividing this product by the number g of nodes in the graph, we obtain exactly the mean number of lines incident with each node. Also in this case, di-graphs present a more complicated situation. There are two kinds of degree for each node: the in-degree, the number of direct lines adjacent *to* a node, and the out-degree, the number of direct lines adjacent *from* a node. It's a simple task to demonstrate that the mean in-degree and out-degree values are equivalent: $\bar{d}_i = \bar{d}_o = L/g$. Nodal degree gives information of how many ties each social actor has; also, we can see for every actor if the relation is direct from each one of them to the other members or the other way round (from this difference we can calculate different centrality indexes: *centrality* and *prestige*). In a social context, the degree indexes can indicate for example the number of friendship ties among a class: if there is a high mean degree we can suppose that in such a class there are many friends; for each subject we can say if his/her friendship is either reciprocated or not (in-degree, out-degree). It might be clear that the meaning of "degree" is different in different contexts of research. Degree in general seen as an index for centrality: in this sense, it can tell us about an actor's prominence in the network (Knoke & Burt, 1983) or his/her prestige (Bonacich, 1987). Furthermore, the mean degree value can be read as a first index of connectivity in our network.

Closely tied to the concept of nodal degree is the second index we're going to present: *density*. When we look at a graph, we observe a set of nodes and a number of lines between them. The density index indicates the proportion of lines really present in the graph over the total number of possible lines (par. 3.2.1). In a graph the formula for computing the density index is $\Delta = L/[g(g-1)/2]$, where L is the number of lines effectively present in the graph and the denominator is the maximum possible number of lines in the graph. In the density formula for di-graphs only the denominator value differs (see par. 3.2.1): $\Delta_d = L/g(g-1)$. The density index takes on a value between 0 (there are no lines/arcs in the graph/di-graph) and 1 (there is the maximum possible number of lines/arcs). Density indicates in an immediate way how our network is cohesive as a whole. It's a simple but fundamental index from which we can start to analyse the structure present in a network (in section 6, we will show some uses of this index, with the aim to describe the structure of the blogosphere, e.g. Adamic & Glance, 2005). The higher is the density index in a graph, the more the actors of our social context are connected each other. We can imagine, for example, our graph as a social support network of a group of old people. The more the graph is dense, the more the social support network connects different individuals to each other member of the group. So, density has been shown to be a good index for group's homogeneity and cohesiveness (Friedkin, 1984).

There is another important "descriptive" index which is easy to calculate, but gives us important information about the closeness of the nodes in the graph: the *diameter.* Defining *geodesic distance* (Harary, 1969) between two nodes the shortest path between them, we define diameter of a graph as the maximum geodesic between any pair of nodes. The diameter value of a connected graph (a graph where any node is reachable by any other node) is defined. When a graph is not connected the diameter value is not defined. The same is in the case of di-graphs. So the diameter index has an important limitation. Many programs used to analyze networks (UCINET, Pajek) calculate two connectivity values based on the number and on the length of geodesics present in a network. The mean geodesic value is calculated using only reachable pairs of nodes. The advantage of these indexes is that they are calculated on connected and not-connected graphs and di-graphs. It's easy to understand that the longer the diameter is, the more a graph is dispersive: if we think our graph to represent a collaborative working group (CWG) in which there a long diameter and/or a high mean geodesic value, we can suppose that communications between different subjects in the CWG may be difficult or at least slow. We will soon explain this discussing Cluster Coefficient.

Heider's (1946) gave the first definition of balance, as this notion involves ties among individual's attitude, opinions or people; he considered direct signed ties.

Harary (1953) introduces the notion of structural balance: let's consider two generic social actors, n_i and n_j, for $1 \leq i \neq j \leq g$; a group is *structurally balanced* when, if n_i and n_j like each other, then they both like and dislike the same other people, or if n_i and n_j dislike each other, then they disagree in every evaluation of all other people.

In a un-equilibrated graph, we could look for some equilibrated subgraphs (that represents, under SNA perspective, equilibrated social actor's sub-networks in a wider social network). The operation that led us to derive the balanced components is called *clusterability*,and every balanced subgraph is called a *cluster* (Davis, 1967).

Holland and Leinhardt (1970) suggested an extension of clusterability to unsigned direct relations: transitivity is the central framework behind their work. They focused on the analysis of triples, and relations (symmetrical, asymmetrical or anti-symmetrical) among any two of them.

A different, and maybe more pertinent to our discussion, use of the term "cluster" is provided by Watts and Strogatts (1998). "*Clustering coefficient*", as in their definition, gives a good measure if a given network can be considered as a "small world" or not.

Let's consider a generic node n_i, with a number $t(n_i)$ of others nodes connected to it (every one of these nodes is called *neighbourhood* of the node n_i), in a connected graph. There can exist at most $\mathrm{t(n_i)(t(n_i)-1)}$ edges between these nodes ($\mathrm{t(n_i)(t(n_i)-1)/2}$ if the graph is not directed). Let C_v denote the fraction of the edges that actually exist in the neighbourhood of a generic node and the allowable edges. The *clustering coefficient C* is defined as the average of every single node in the graph C_v. These two authors show that a highly clustered network has the characteristics of a small world network. For our purpose, this index can be truly determinant: many researches on the structure of the web, like those we will show in section 6, try to discover if the web (or simply, the blogosphere for a specific argument) can be considered a small world. With these indexes, we can draw some interesting conclusions.

The aim of this paragraph was to present how Graph Theory has been used in Social Network Analysis. To do this, we have presented the fundamental indexes necessary for a first analysis, which are also the ones from which all the researches (both formal and empirical) about the relations between these two theories, started.

4. Algebraic models

Another class of models for analysing social networks comes from an algebraic perspective, in particular from algebra of semigroups and order theory. The rationales for this approach are two: the first one is the opportunity of using profitable models "that will enable the analysis of network data in a number of different forms". A second theme is the possibility to develop "data models attuned to variety of theoretical claims about the nature of the role of social networks in the development of social and psychological relations" (Pattison, 1993, XIX).

Lorrain and White (1971) emphasize the primary importance of the concept of role in a group: role of an individual in a social system "has often been described as consisting of sets of relations of various types linking this person as ego to sets of others" (p.350). So, the *role* of an individual is identified by the incoming and outcoming ties this individual has with other members of the group. Tools proposed to identify and describe a role, different roles in a social network, and a way to compare social structure considering different role structures, come from semigroup algebra, as proposed by White.

The starting point is the notion of *Structural Equivalence*:

Definition 3: Let R = $\{R_1, ..., R_p\}$ be the set of relations in a network **W**. Two individuals x and y are said to be Structurally Equivalent if, for all nodes $k = 1, \ldots, t$ ($k \neq x,y$), xRk iff yRk, and kRx iff kRy.

That is, two nodes are structurally equivalents if and only if they share the same set of ingoing and outgoing ties with a defined individual. If two actors are structurally equivalent, they are said to hold the same position in the network (White et al., 1976). Individuals sharing the same position can jointly form a partition of a group: this kind of partition is said to be a "blockmodel", and it is representative of the social position that each block member holds. A blockmodel may be viewed as a new Social network, in the sense that it can be represented as a set of multiple binary relations defined among blocks.

Some criteria for determining a fit to a blockmodel of a set of data with a weaker condition than structural equivalence has been proposed. One, called *lean-fit*, is a weaker condition of the *fat-fit*, in the sense that at least one individual in a block has a relation to every individual in the second (Breiger et al., 1975). Another, called α–

blockmodel fit, is a condition where only a limited number of relations (the number of total relation multiplied by α, called *criterion*) is considered (Arabie et al., 1978).

One of the most remarkable consequences of the use of a blockmodel is that roles (and role structures) are not a-priori defined, but are seen as emergent and elastic structures to describe a social group: a role is completely defined by the set of relations composing it; furthermore, an individual could be unaware of his role.

We will now describe, with the aid of an example, how to build an algebra for a social network. This description is given for two main reasons: one is to illustrate how it is possible to compare different networks, and to have some further information about network structure. The other is that we want to describe, at a very simple level, how an algebraic analysis is made, in order to give an introduction to this complex topic.

We will limit our example to a complete network (we mean complete in the sense described in section 2.4); although building an algebra for an ego-centred network is similar to the construction of a complete network, there are still some differences and difficulties, which are not in our interest to explore in this paper. For further details, a complete description of a local role algebra construction, that is the algebra made from an ego-centred network, is provided in Pattison (1993).

The basic notion of algebraic modelling is *composition* (among binary relations): according to Lorrain and White (1971), composition represents “the basic logic of interlock in the system of relationship”((Lorrain & White, 1971, in Pattison, 1993, p. 40). Pattison writes that “compound relations are claimed to define the paths where social processes flow”. Lorrain (1972) says that “any concatenation of social relationship is itself a social relationship, whether perceived or not” (Lorrain, 1972, in Pattison, 1993, p. 41). By composing two (or more) primitive relations, we may produce a collection of compounds relations.

Let’s consider the two relations already discussed in section 3.2. If we compose a relation A with a relation B, say AB, we will have a new kind of relation (see table. 3) (the operation of composition is not commutative, i.e. AB is not necessarily the same as BA).

$AB = \{(x_1, x_1), (x_2, x_1), (x_3, x_2), (x_3, x_3), (x_4, x_1), (x_4, x_4)\}$

	x_1	x_2	x_3	x_4
x_1	1	0	0	0
x_2	0	1	0	0
x_3	0	1	1	0
x_4	1	0	0	1

Table 3: The list of the couples composing relation *AB*, and the corresponding Boolean matrix.

There are an infinite number of possible compound relations (for instance, AB, BA, AAB, BBA, AAABBAAA and so on). So, a way to compare relations is needed.

We can consider if a relation is included in another relation (i.e. if a relation is a subset of another relation). We can see, in our example, that every couple in *B* is also in *A*, but not the other way round. So, *B* is a subset of *A*; we write $B \leq A$. As stated from the anti-symmetric property of inclusion, if A and B are binary relations on a set X, we define $A = B$ if and only If $A \leq B$ and $B \leq A$. If two or more relations (primitives or compound) are equal, they are redundant so one can be expressed by the other (Boorman & White,1976).

All distinct relations can be found by an iterative process (the algorithm is described in Pattison, 1993, p. 49).

Using the relation ≤, it's possible to build an order among primitive and compound relations, with the following three properties: reflexive, transitive and anti-symmetric ($A \leq B$ and $B \leq A$ imply $B = A$, for every *A*, *B* in *X*). In mathematics, this order is called "partial order".

A	B	AA	AB	BA
0 1 0 0	0 1 0 0	1 0 0 0	1 0 0 0	1 0 0 0
1 0 0 0	1 0 0 0	0 1 0 0	0 1 0 0	0 1 0 0
1 0 0 1	0 0 0 1	0 1 1 0	0 1 1 0	0 1 1 0
0 1 1 0	0 0 1 0	1 0 0 1	1 0 0 1	1 0 0 1

BB	BBA	BBB	BBAA	BBBB
1 0 0 0	1 0 0 0	0 1 0 0	1 0 0 0	1 0 0 0
0 1 0 0	0 1 0 0	1 0 0 0	0 1 0 0	0 1 0 0
0 0 1 0	0 1 1 0	0 0 0 1	0 1 1 0	0 1 1 0
0 0 0 1	1 0 0 1	0 0 1 0	1 0 0 1	1 0 0 1

Table 4: Some of the compound relations of *A* and *B*. As one can see, A = BBA ≤ B = BBB, BB ≤ AA = AB = BA = BBAA = BBBB. So, to describe the possible relations on the network is sufficient to use relation A, B, AA, and BB (every other relation is congruent with one of these). The partial order is A ≤ B, BB ≤ AA, A and BB, A and AA, B and AA, B and BB are not comparable.

We will now describe the main concept, the *partially ordered semigroup*. A semigroup is an algebraic structure defined by a domain *X* and a characteristic binary operation *O* among members of *X*, so that *O* is internal in *X* and is associative.

If we consider every relation (primitive and compound) in the network as an element, and the composition operation as the characteristic operation of the structure, a semigroup is obtained. Now we can create a representation of the network as a “partially ordered semigroup” (see table 5). Why is it so useful to build a semigroup? Because it gives an abstract representation of a network, which is possible to compare with other abstract representations (i.e. another semigroup), in order to find differences and similarities between different networks. Let’s consider the two partially ordered semigroups of networks $\boldsymbol{N_1}$ and $\boldsymbol{N_2}$, respectively $S(\boldsymbol{N_1})$ and $S(\boldsymbol{N_2})$. If there exist some mappings so that every element of $S(\boldsymbol{N_1})$ can be transformed into an element $S(\boldsymbol{N_2})$, the entire structure of $S(\boldsymbol{N_1})$ can be replaced by the structure of $S(\boldsymbol{N_2})$. The two structures are said to be structurally equivalent, and that they share the same skeleton. A network and its blockmodel share the same skeleton.

	Composition				Partial order			
Element	A	B	AA	BB	A	B	AA	BB
A	AA	AB	A	A	1	1	0	0
B	AA	BB	A	B	1	0	0	0
AA	A	A	AA	AA	0	1	0	0
BB	A	AB	AA	AA	1	1	0	0

Table 5: This table shows the semigroup obtained from the network in table 1, and the partial order on it. The composition section must be read in this way: if we compose the element in the row with the correspondent element in the column, result is the relation in the correspondent cell. The partial order section must be read in this way: if there is the relation ≤ from the element in the row to the correspondent element in the column, value is 1, otherwise is 0.

It's possible to operate more complex comparisons among semigroups, but it's not our aim to describe that. For every detail, a global description of methods of analysis of semigroups is provided by Pattison, 1993, Pattison & Bartlett, 1982, Pattison & Wasserman, 1995, Pattison et al., 2000.

5. Dynamic Network Analysis (DNA) and statistical issues

In this section two fundamental aspects of SNA will be exposed: the first represents a new development of the complexity of the theory (DNA), while the second is a topic always present in SNA, that in the last years has become one of the most interesting fields of research in SNA (statistical issues).

One of the most recent developments in SNA considers network as an ever-changing and dynamic structure: this approach is called Dynamic Network Analysis (DNA).

While SNA concentrates mainly on small, bounded networks, on a limited and sometimes small number of relations, observed only at one point in time, DNA tries to study multi-modal, multi-plex[1] dynamic networks with varying level of uncertainty. The aim of this second kind of studies is to understand how a network can evolve, change and stabilize (and eventually, which kind of intervention could be done in order to modify a network in a chosen direction).

Some of the main peculiarities of DNA are the use of a meta-matrix of relation, the treatment of ties as probabilistic and the combination of social network with cognitive sciences and multi-agent systems (Carley, 2003).

A meta-matrix (Carley, 2002) is a multi-colour and multiplex representation of nodes and the connections among them. Considering a group of people, not only relations among them are considered, but also relations in the field of the global information inside the network, people's needs, linkages among every organization they are in and so on. The fundamental idea is that a change in one network can cause a change in the others, because every relation implies a relation in another one.

Probabilistic ties are considered because in this kind of network not all the information about the structure is well-known; probability of a relation can be affected by factors such as the observer's certainty in a tie or the likelihood that a given tie is manifest at a moment in time.

Multi-agent system means that social actors are treated as active adaptive agents capable of taking action that can alter the network structure. Social and cognitive processes that can influence ties are considered.

As stated in Wasserman and Faust, in 1993, "the statistical approach has been in use since the beginnings of social network analysis. However, it was not widely used until the research of Holland and Leinhardt (1970, 1971)" (Wasseman & Faust, 1994, p. 506).

The fundamental concept is that of *random graph*: a random graph is a graph in which properties such as the number of graph vertices, graph edges, and connections between them are determined in some random way. Some distribution models have been proposed, from the simple Conditional Uniform Distribution (Katz & Powell, 1957) to exponential family of Probability Distributions (Holland & Leinhardt, 1981). Wasserman & Iacobucci (1988) propose a generalization of the last distributions, based on log-linear models; in the same paper, two statistical models are proposed: an associative one, that allows the study of changes of a network over time, and a predictive one, that permits to predict the state of a network in a time point if the previous states are known.

[1] Multi-plex ties are those in which several types of relationships come together.

A dependence graph considers the statistical dependencies among the elements of random variables based on the relational ties in a network: it is a graph in which nodes are the set of all possible ties, and edges, specifying the pairs of ties, are conditionally independent (Wasserman & Robins, 2005)

Some random graph models for multiple relations networks are described by Koelhy and Pattison, 2005:

Statistical analysis has been used in order to build algebraic models (Pattison & Wasserman, 1995), to know the tendency over reciprocation of choice, or mutuality (Katz & Powell, 1955), to study network evolution over time (Snijders, 2005), to compare sociometric relations recorded in distinct sociomatrices (Wasserman, 1987), and so on. Statistical approach now represents one of the most productive and ongoing fields in SNA.

6. Social Networks and ICT (Information and Communication Technology).

It's quite simple to understand SNA potentialities in the study of social aspects of the web. Maybe the first study on Computer Mediated Communication (CMC) was made by Freeman (1986): he used a dataset consisting of measurements of computer mail interaction, in order to study how this computer network modified acquaintenships and friendship. Park (2003) distinguishes among various types of networks: a social network is defined as a set of people connected by a set of relationships, where relationships are any kind of social relation; a communication network is composed by interconnected individuals linked by patterned flows of information. This classification proceeds presenting three other kinds of networks that are more related to the content of this section: computer mediated network, a kind of communication network where all the relations are mediated by computer; internet network is a computer mediated network which uses internet as channel of information flow; hyperlink network is an internet network that uses hyperlinks as channel of information flow. As we can see, among any kind of network there is a relation of inclusion (a kind of network is included in another one, where SNA is the most general case).

In the last years, researches in this field have looked with increasing interest to the blogosphere phenomenon using the hyperlink networks. Herring et al. (2005) have investigated the structure subtended by a sample of 5517 blogs about religion. They have looked at some SNA indexes taken from Graph Theory like in-degree, out-degree, reciprocity and so on. Their findings were that some blogs are overrepresented and central in the network, although other blogs are more densely interconnected. Majority of blogs link sparsely or not at all to other blogs in the sample, suggesting that the blogosphere is partially interconnected and sporadically conversational. Adamic & Glance (2005) analyzed a list of 40 political blogs (conservative or liberal) for a 2 month period and a sample of 1000 political blogs for a day in the period of USA presidential election in 2004. They tried to investigate the relations between the two groups of blogs and to discover the structure subtended by the two groups and possible differences between them. They used some measures of SNA like in-degree and out-degree and they found that liberals and conservatives link primarily within their separate communities, with far fewer cross-links exchanged with the other party. Conservative bloggers more likely link to other blogs: primarily other conservative blogs, but also some liberal ones. The conservative blogosphere is more densely linked. Starting from these results, Ackland (2005) proposes an interesting distinction between 2 types of blogs: *authorities* are highly-referenced pages on a particular topic, while *hubs* are pages that point to the authorities (and thus confer authority). There is a mutually reinforcing relationship between authorities and hubs: a good hub points to many good authorities, and a good authority is pointed to by many good hubs.

On one hand we can see how SNA is a very suitable tool to analyze the structure (if there is any) of the blogosphere; on the other hand the analysis of blogosphere, thanks to its peculiarities, provides many interesting cues for the theory evolution. Such a kind of topological analysis of linkage structure underneath websites, the characteristics of this structure and the comparison between different sets of web entities are only possible using the tools provided by SNA.

Sometimes when SNA is used to study social structures in the web and the investigated relation is only hyperlinking, we refer to it as Hyperlink Network Analysis (HNA). HNA has been used in several different fields of research, like e-commerce (Park et al., 2001; Palmer et al., 2000; Krebs, 2000) or international communication (Barnett et al., 2001; Hargittai, E., 1999).

An interesting example is provided by Adamic and Adar (2003): they considered home pages and mailing lists of MIT and Stanford University students to investigate the presence of any factors which indicate social connections between individuals. Classical SNA indexes like in and out-degree are here employed in addiction to some peculiar indexes like *clustering coefficient* (ratio between couples of individuals linking each other and the total possible couples, where a high value of this index indicates the presence of a small world) and *cohesiveness of subgroups associated with a data item*, obtained by the ratio between total and within-group density.

From a larger perspective, if we consider internet network (as defined in the beginning of this section), Paolillo (1999) analyzed the modifications of common language in a virtual speech community trying to identify a correlation between roles in the network and the language modifications used. In other words, the aim of this paper is to demonstrate that the more a node (a group of subjects in the community in this case) is central and have strength ties with the others, the more some language modifications, typical in a IRC, are concentrated around this group.

Some studies have explored the behaviour of users inside networks, and not only the structure: Adamic and Adar (2005) has tried to investigate how people are able to select among hundred of acquaintances the correct person to form the link in the chain. Two different kinds of network have been examined: one is that of email mutual contacts inside an Information Technology group, and one is built upon some buddy list of Stanford students. As one could expect, it's easier to reach a person in a network when the network is hierarchically organized and well defined and its structure is well known to each member of the network.

Licoppe and Smoreda (2005), starting from some previous data, advanced some reflections on the relation between social networks and Technology (in the larger sense of this term). These reflections were inserted in a broader analysis context of how technologies can influence communication finalities and modalities. It seems very interesting over all the finding that the most recent media (like SMS or Instant Messaging Softwares) are often centred on the *phatic* function of communication (as intended by Jakobson, 1960). The discursive content of the communication gestures results less important than the message itself, and how, instead of the play between absence and co-presence (as in traditional phone mediated communications), we would have a play between lack of attention and absorption, between safety and interactional vulnerability. "*The risk is that ties with friends will become institutionalised in the form of expectations and mutual obligations to be constantly available electronically"* (Licoppe & Smoreda, 2005).

In the last years CMC has become a fundamental tool for several different fields of research and knowledge. SNA is a useful tool to understand how subjects interact with one another through CMC. The application field of CMC is really huge but there are some areas in which it seems to be particularly advanced: Cho, Stefanone and Gay (2002) try to clarify how SNA can be very useful in order to analyse the development of a Computer Supported Collaborative Learning (CSCL) group; in a similar framework, the research of Palonen and Hakkarainen (2000) aimed to investigate, using SNA, patterns of elementary school students' peer interaction in a CSCL environment, which in this case was a computer supported classroom; a very interesting work has been made by Martinez et al.(2003), using CSCL, the authors propose a mixed approach combining traditional data sources with computer logs and integrating quantitative statistics, qualitative data analysis and SNA to provide an overall interpretative approach to the explanation of classroom social interactions.

In section 5, we briefly introduced the concept of Dynamic Network Analysis. DNA has shown to have a wide range of applications in Information Technology related fields: for example, it has been usedin the analysis of factors affecting team successes and failures in videogames (Carley et al., 2005).

A very common problem in studying Social Networks on the web is that of data retrieval. As Park (2003) pointed out, there are two ways to measure hyperlinks as relations: observation and computer-assisted measurement. We find that direct observation has been the most employed measurement technique for gathering this kind of data (Herring, 2005). The problems associated with this method are in the possibility of coding errors and the high labor cost (the researcher has to surf among a huge number of sites). On the other side, the computer assisted tools up to now developed are "*idiosyncratic answering only the research question being investigated"* (Park, 2003).

7. Conclusions

As we have just pointed out at the end of section 2.1, SNA has been seldom criticized for concentrating more on methodological issue than on the empirical one (Barnes & Harary, 1983). In this paper, we have tried to show how the formal aspects of this topic are complex and advanced, basing on strong and consolidated mathematical theories, such as graph theory and semi-groups algebra, and what the potentialities and the benefits of this approach are. Many methodological problems, if concerning the possibility to study social aspects of the web, are still to be solved, mainly in sampling. For example, in Herring (2005), a snowball procedure has been used: in a wide and complex relation network, constituted by a huge number of nodes (as in blogosphere, a growing, complex and with not well defined boundaries domain), sometimes this procedure could hide the real structure beneath the network, because it considers only a limited number of links and composes a map that not necessarily corresponds to the real one.

Also, the real meaning of a hyperlink as a communication relation, and what the meaning of social network indexes when applied to web is, have to be investigated.

Notwithstanding these problems, Social Network Analysis could be a powerful and useful tool for examining the social aspects on the web. As we have shown in section 6, in the last years we have seen a growing number of studies on this topic. Many aspects should be still examined: for example, SNA indexes would be suitable for examining online gaming communities and their behaviour. Such a community would be easier to examine, because of a simpler sampling procedure, and the possibility to study relations also different form hyperlinking. Another field with many potentialities is that of Instant Messaging Software: previous research, as that shown in section 4, describes this network as a spare and chaotic network. Some constraints could be applied to the modalities of Instant Messaging, to give a more defined structure (for example, to help with the search of an individual as described in Adamic & Adar, 2005).

Potentially, these new ways of communication could offer the possibility to measure relations in a dynamic way, with the possibility given from SNA to offer some description of how networks work and evolve. If a group communicates by computer-mediated instruments, the analysis of role structures (as described in section 3.3) could permit the study of complex structures, that define the group and have not been yet studied (for example, how computer mediated communication can change the role structure of a group). To our knowledge, there are no studies using Social Network Analysis perspective to explore Internet or Computer Mediated Communication with the aim to study the social structure of a group as described in section four. This is maybe due to the fact that this kind of tools are significant only in the case of a multi-relational network, while we know, for example, that a hyperlink network is constructed only on the basis of one relation. We think, for example, that role algebra could be a powerful tool to explore the difference and the structures of networks where actors use different communication instruments to relate to each other. A relation would be constructed on the basis of the used instrument.

As outlined in the famous paper by Barnes and Harary (1983), possibilities also arise from a mutual exchange between SNA and ICT: new fields of applications imply the possibility for SNA to improve the empirical applications of the theoretical findings (for example, the sampling problem described in these conclusions), with the aim to study different and more common (in the literature) kinds of networks.

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